1.7 - Venn Diagrams will be covered simutaneously with 1.5 and 1.6.

1.5 - Union, Intersection, and Difference

There is a so-called "algebra of sets" whose operations are given by unions $\cup$, intersections $\cap$, and differences $-$.

Def: Let A and B be sets. Define:

the union of A and B:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

the intersection of A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

the difference of A and B

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

Note $A - B$ is also often written $A \setminus B$.

Facts:
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$
$$A - B \neq B - A$$

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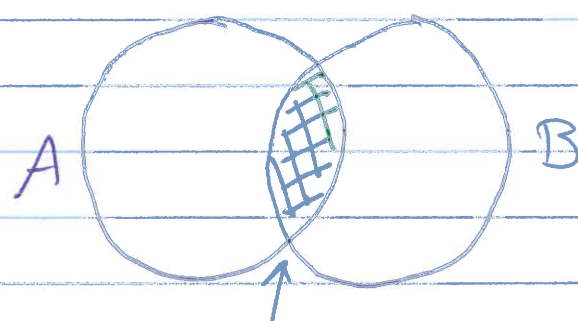
It's easiest to use Venn diagrams to get a conceptual understanding of what these things mean. In this way, we imagine the sets as circles, and we shade in the regions corresponding to the set operation.

Union: $A \cup B$

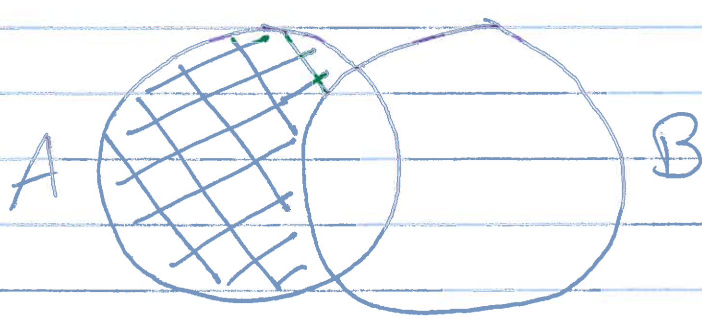
$A \cup B$

Intersection: $A \cap B$



$A \cap B$

Difference: $A - B$



Ex: Let $A=\{1,2,3,4,5\}$, $B=\{4,5,6,7\}$
$C=\{5,6,7,8,9\}$, $D=\{1,3,5,7,9\}$
$E=\{2,4,6,8\}$, $F=\{1,5,9\}$
$\mathcal{U}=\{1,2,3,4,5,6,7,8,9\}$

(a) $A\cup B=\{1,2,3,4,5,6,7\}$
$A\cap B=\{4,5\}$
(b) $B\cup D=\{1,3,4,5,6,7,9\}$
$B\cap D=\{5,7\}$
(c) $A\cup C=\{1,2,3,4,5,6,7,8,9\}=\mathcal{U}$
$A\cap C=\{5\}$
(d) $D\cup E=\{1,2,3,4,5,6,7,8,9\}=\mathcal{U}$
$D\cap E=\emptyset$
(e) $E\cup E=\{2,4,6,8\}=E$
$E\cap E=\{2,4,6,8\}=E$
(f) $D\cup F=\{1,3,5,7,9\}=D$
$D\cap F=\{1,5,9\}$

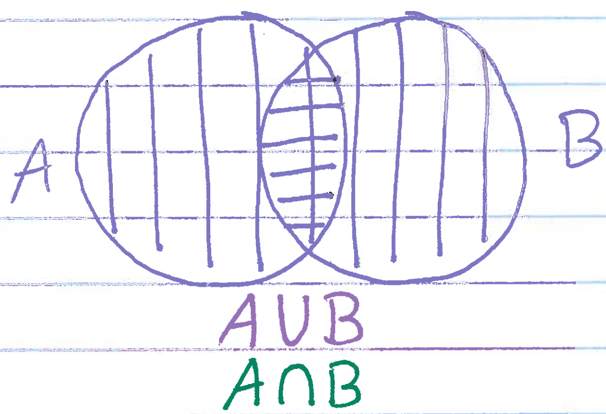
Facts (1) For any set A, $A\cap A=A\cup A=A$

(2) If $A\subset B$, then $A\cap B=A$
$A\cup B=B$

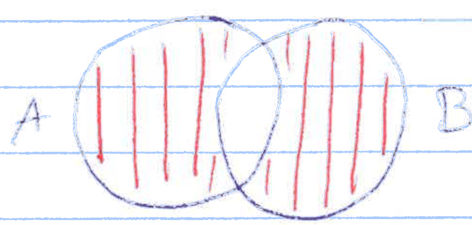
Ex: a) $A - B = \{1, 2, 3\}$
$B - A = \{6, 7\}$
b) $D - E = \{1, 3, 5, 7, 9\} = D$
c) $F - D = \emptyset$

Ex: Draw a Venn diagram for the sets:
a) $(A \cup B) - (A \cap B)$
b) $(A - B) \cup (B - A)$

a)

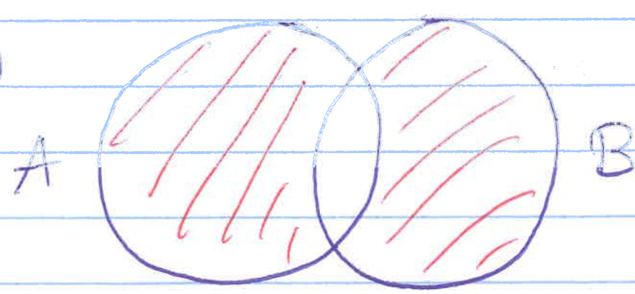


So



$(A \cup B) - (A \cap B)$

b)



$(A - B) \cup (B - A)$

This suggests $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

This is true, and we call this set the <u>symmetric difference</u>:

$$\boxed{A \triangle B = (A-B) \cup (B-A)}$$

1.6: Complements

Suppose we have the set $T = \{1, 2, 4, 8, 16, 32, \ldots\}$. What are elements not in T?

You might naturally say things like 3, 5, or 10, but would probably not immediately think of π or a mouse. We typically think of T as a subset of $\mathbb{N}$. For this reason, we would call $\mathbb{N}$ the <u>universal set</u> in this situation. (Of course, we could specify a universal set to be something else if we wanted/needed to.) This gives us a natural notion of the "outside" or complement of a set:

<u>Def</u>: Let A be a set with universal set $\mathcal{U}$. The <u>complement</u> of A, denoted $\overline{A}$, is the set

$$\boxed{\overline{A} = \mathcal{U} - A}$$

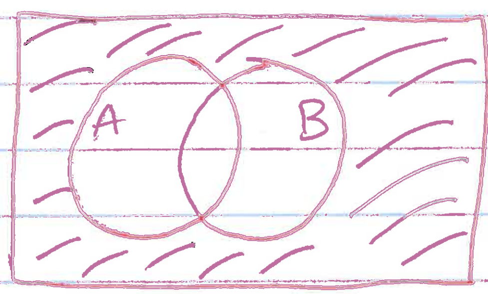
* Often $\overline{A}$ is written as A^c since the bar usually has some other meaning.

Ex: $\overline{T} = \{3,5,6,7,9,10,11,12,13,14,15,\dots\}$
$= \{x \in \mathbb{N} \mid x \neq 2^n, n \in \mathbb{N} \cup \{0\}\}$

DeMorgan's Law

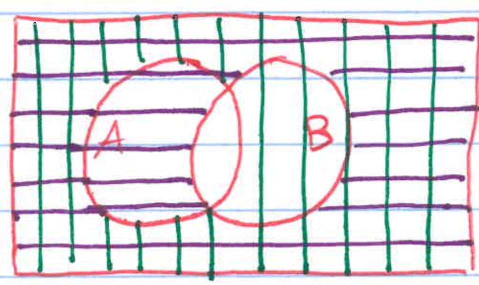
For sets A, B i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Proofish of (i)

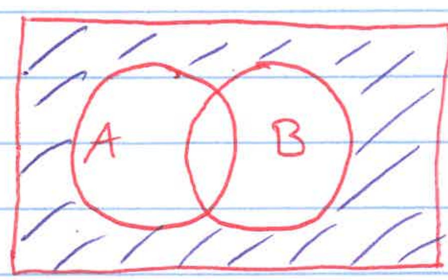


$\mathcal{U}$

$\overline{A \cup B}$



$\underset{|||}{\overline{A}} \cap \underset{\equiv}{\overline{B}} = \#$



$\overline{A} \cap \overline{B} = \overline{A \cup B}$

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<u>Facts</u>: If $\mathcal{U}$ is a universal set

(a) $\overline{\mathcal{U}} = \emptyset$

(b) $\overline{\emptyset} = \mathcal{U}$